Hong Kong Mathematics Olympiad (2000 – 2001)

Heat Event (Group)

香港数学竞赛 (2000 – 2001)

初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 现在钟面上的时间是一时正。 p 分钟后，分针与时针刚好重迭，求 p 的最小值。

   The time on the clock face is now one o'clock. After p minutes, the minute hand overlaps with the hour hand, find the minimum value of p.

2. 把 10 个完全相同的球放入 3 个不同的盒子里，使得没有一个盒子是空的，共有多少种放法？

   In how many ways can 10 identical balls be distributed into 3 different boxes such that no box is to be empty?

3. 设 $x=\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}$ 及 $y=x^2$，求 y 的值。

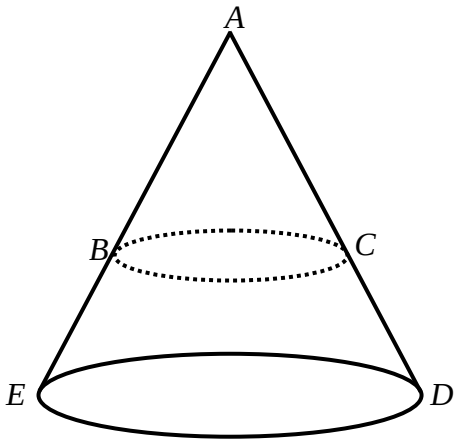
   Let $x=\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}$ and $y=x^2$, find the value of y.

4. 如果 $\dfrac{4a}{1-x^{16}}\equiv\dfrac{2}{1-x}+\dfrac{2}{1+x}+\dfrac{4}{1+x^2}+\dfrac{8}{1+x^4}+\dfrac{16}{1+x^8}$，求 a 的值。

   If $\dfrac{4a}{1-x^{16}}\equiv\dfrac{2}{1-x}+\dfrac{2}{1+x}+\dfrac{4}{1+x^2}+\dfrac{8}{1+x^4}+\dfrac{16}{1+x^8}$, find the value of a.

5. 如图一，ADE 是一个直立圆锥体。如果从底部向上并在 $\dfrac{1}{4}$ 的高度平行底部横切，上面细锥体 ABC 斜面与余下底部 $BCDE$ 斜面的面积的比为 $1:k$，求 k 的值。

   In Figure 1, ADE is a right circular cone. Suppose the cone is divided into two parts by a cut running parallel to the base and made $\dfrac{1}{4}$ of the way up , the ratio of the slant surface of the small cone ABC to that of the truncated base $BCDE$ is $1:k$, find the value of k.



图一

Figure 1

6. 如果十位数 $2468m2468m$ 可被 3 整除，求 m 的最大值。

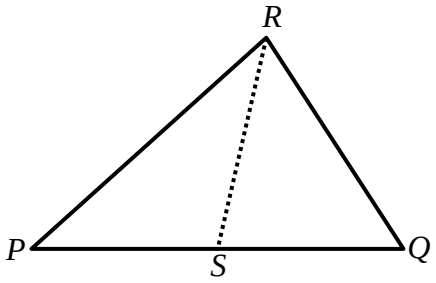
If a ten-digit number $2468m2468m$ is divisible by 3, find the maximum value of m.

7. 求由 x-轴 及直线 $x-3y=0$、$x+y-4=0$ 围出的面积。

Find the area enclosed by the x-axis and the straight line $x-3y=0$, $x+y-4=0$.

8. 如图二，PQR 是一个三角形，S 是 PQ 上的中点，$RQ=PS=SQ$，且 $\angle RQS=2\angle RPS$。设 $\angle PSR=x°$，求 x 的值。

In Figure 2, PQR is a triangle, S is the mid-point of PQ, $RQ=PS=SQ$, and $\angle RQS=2\angle RPS$. Let $\angle PSR=x°$, find the value of x.



图二

Figure 2

9. 如果 x 满足方程 $|x-3|+|x-5|=2$，求 x 的最小值。

If x satisfies the equation $|x-3|+|x-5|=2$, find the minimum value of x.

10. 从 6 对不同型号的鞋子中任取 3 只，求 3 只鞋子中恰有 2 只是同一型号的概率。

3 shoes are chosen randomly from 6 pairs of shoes with different models, find the probability that exactly two out of the three shoes are of the same model.